

## Exercise 305

According to the World Bank, at the end of 2013 ( $t = 0$ ) the U.S. population was 316 million and was increasing according to the following model:  $P(t) = 316e^{0.0074t}$ , where  $P$  is measured in millions of people and  $t$  is measured in years after 2013.

- Based on this model, what will be the population of the United States in 2020?
  - Determine when the U.S. population will be twice what it is in 2013.
- 

### Solution

#### Part (a)

2020 is 7 years after 2013, so plug in  $t = 7$  to the equation.

$$P(7) = 316e^{0.0074(7)} \approx 332.80$$

According to the model, the population will be about 333 million in 2020.

#### Part (b)

Double the population of 316 million is 632 million.

$$P(t) = 316e^{0.0074t}$$

$$632 = 316e^{0.0074t}$$

Divide both sides by 316.

$$2 = e^{0.0074t}$$

Take the natural logarithm of both sides.

$$\ln 2 = \ln e^{0.0074t}$$

Use the property of logarithms that allows the exponent of the argument to be brought down in front.

$$\ln 2 = (0.0074t) \ln e$$

Use the fact that  $\ln e = 1$ .

$$\ln 2 = 0.0074t$$

Solve for  $t$  by dividing both sides by 0.0074.

$$t = \frac{\ln 2}{0.0074} \approx 93.67$$

Therefore, it will take about 94 years from the end of 2013 for the population to double.